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E_3 - CORDIAL LABELING AND TOTAL 3 - SUM CORDIAL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF ARROW GRAPH

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Abstract: In this paper, we prove that the extended duplicate graph of arrow graph admits total 3 sum cordial, E_3-cordial and total E_3- cordial labeling.

AMS subject classification: 05C78

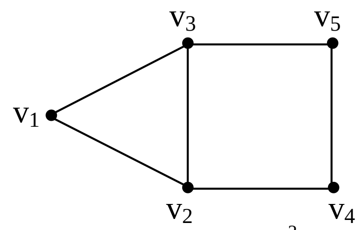
Keywords: Duplicate graph, arrow graph E_3-cordial, total 3 sum cordial.

1. Introduction: All graphs in this paper are finite, simple and undirected G(V,E),with $|V| = p$ vertices and $|E| = q$ edges. For all terminology and notations we follow Harary [4]. Most of the graph labeling techniques trace their origin to graceful labeling introduced independently by Rosa [6] and Golomb[7].The concept of duplicate graph was introduced by E.Sampthkumar and he proved many results on it [2].For an extensive survey on graph labeling and bibliographic references we refer to Gallian[3]. K.Thirusangu, B.Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [8]. In 2000, Cahit and Yilmaz[1] introduced the concept of E_3 Cordial labeling. V.K.Kaneria, M.M.Jariya and H.M.Makadia[9] discussed the idea of arrow graph. S.Pethanachi Selvam and G.Lathamaheswari[5] studied total 3-sum cordial labeling of some graph.

2. Preliminaries: In this section, we give the basic definitions relevant to this paper. Let G(V,E) be a finite, simple and undirected graph with p vertices and q edges.

Definition 2.1 Arrow Graph: An arrow graph A_m^n with width 'n' and length 'm' is obtained by joining a vertex 'v' with superior vertices of $P_t \times P_m$ by ' t' new edges from one end. Clearly it has 2m+1 vertices and 3m edges.

Illustration 1: ARROW GRAPH

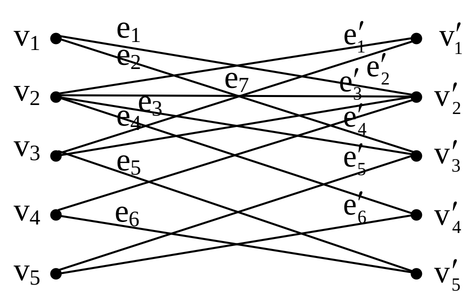


A^2_2

Definition 2.2 Duplicate Graph: Let G (V,E) be a simple graph and the duplicate graph of G is DG (V_1, E_1) ,where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f: V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.

Definition2.3 Extended duplicate graph of Arrow graph: Let DG (V_1,E_1) be a duplicate graph of the arrow graph G(V,E). Extended duplicate graph of arrow graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph. It is denoted by EDG (A^2_m). Clearly it has 4m+2 vertices and 6m+1 edges, where 'm' is the number of length.

Illustration 2: EXTENDED DUPLICATE ARROW GRAPH



EDG (A_2^2)

Definition2.4 Total 3 sum cordial labeling: Let f be a map from $V(G)$ to {0, 1, 2}.For each edge uv, assign the label $(f(u)+f(v))(\text{mod}3)$.The map f is called Total 3 sum cordial labeling of G,if $|f(i)-f(j)|\leq 1, i\neq j$ and $i, j\in\{0,1,2\}$

Definition2.5:E_3 cordial labeling: Let G be a graph with vertex et V and edge set E.An edge labeling $f:E\rightarrow Z_3$ where $Z_3=\{0,1,2\}$ induces a vertex labeling $f^*:V\rightarrow Z_3$ defined by $f^*(v_i)=\{\sum f(v_iv_j)/v_iv_j\in V\}(\text{mod}3)$ for all vertex $v\in V$ .For $i\in Z_3$,let $m_i(f)=\{e\in E/f(e)=i\}$ *and* $n_i(f)=\{v\in V/f^*(v)=i\}$. If $|m_i(f)-m_j(f)|\leq 1$ and $|n_i(f)-n_j(f)|\leq 1, i\neq j \forall i, j\in Z_3$, it is called E_3-cordial labeling.

Definition2.6: Total E_3 cordial labeling: An E_3-cordial labeling f is said to be total E_3-cordial labeling of G if for all $i, j\in Z_3: |\{m_i(f)+n_i(f)\}-\{m_j(f)+n_j(f)\}|\leq 1.$

3. MAIN RESULTS:

3.1 TOTAL 3 SUM CORDIAL LABELING

In this section, we present an algorithm and prove the existence of total 3-sum cordial labeling for the extended duplicate graph of arrow graph A_m^2 , m ≥ 2.

Algorithm: 3.1

Procedure [Total 3 Sum Cordial labeling for EDG (A_m^2), m≥2]

$V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v_{2m+1}, v'_1, v'_2, \ldots, v'_{2m}, v'_{2m+1}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{3m}, e_{3m+1}, e'_1, e'_2, \ldots, e'_{3m}\}$

$v_1\leftarrow 1, v_4\leftarrow 1, v_2\leftarrow 2, v_3\leftarrow 0$
$v_1'\leftarrow 1, v_2'\leftarrow 2, v_3'\leftarrow 0$

for $i=0$ *to* $\left\lfloor\frac{m-2}{3}\right\rfloor$ ***do***

$v_{5+6i} \leftarrow 0$

end for

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-3}{3} \right\rfloor$ **do**

$v_{7+6i} \leftarrow 2$

$v_{6+6i} \leftarrow 0$

end for

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-4}{3} \right\rfloor$ **do**

$v_{9+6i} \leftarrow 1$

$v_{8+6i} \leftarrow 2$

end for

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-5}{3} \right\rfloor$ **do**

$v_{10+6i} \leftarrow 1$

end for

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-2}{3} \right\rfloor$ **do**

$for\ \ j = 0\ \ to\ 1$ **do**

$v'_{4+6i+j} \leftarrow 2$

end for

end for

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-3}{3} \right\rfloor$ **do**

$for\ \ j = 0\ \ to\ 1$ **do**

$v'_{6+6i+j} \leftarrow 1$

end for

end for

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-4}{3} \right\rfloor$ **do**

$for\ \ j = 0\ \ to\ 1$ **do**

$v'_{8+6i+j} \leftarrow 0$

end for
end for
end procedure

Theorem 3.1: The extended duplicate graph of arrow graph A_m^2, m ≥ 2 is total 3 sum cordial.

Proof: Let A_m^2, m ≥ 2 be a arrow graph**.** In order to label the vertices, define a function *f*: V → {0,1,2} as given in algorithm 3.1.

The vertices v_1, v_2, v_3, v_4, v'_1, v'_2 and v'_3 receive label '1' , '2' , '0' , '1' , '1','2' and '0' respectively **;**

for $0 \le i \le \lfloor (m-2)/3 \rfloor$, the vertices v_{5+6i} receive label '0' ; for $0 \le i \le \lfloor (m-3)/3 \rfloor$, the vertices v_{6+6i} receive label '0' and the vertices v_{7+6i} receive label '2' ;

for $0 \le i \le \lfloor (m-4)/3 \rfloor$, the vertices v_{9+6i} receive label '1' and the vertices v_{8+6i} receive label '2'; for $0 \le i \le \lfloor (m-5)/3 \rfloor$, the vertices v_{10+6i} receive label '1'

for $0 \le i \le \lfloor (m-2)/3 \rfloor$ and $0 \le j \le 1$,the vertices v'_{4+6i+j} receive label '2' **;** for $0 \le i \le \lfloor (m-3)/3 \rfloor$ and $0 \le j \le 1$, the vertices v'_{6+6i+j} receive label '1' **;** for $0 \le i \le \lfloor (m-4)/3 \rfloor$ and $0 \le j \le 1$,the vertices v'_{8+6i+j} receive label '0' .

Thus, the entire 4m+2 vertices are labeled.

To obtain the labels for edges , we define the induced function f * : E → {0,1,2} such that f * ($v_i v_j$) = {$f(v_i)$+ $f(v_j)$ }(mod 3) where v_i, v_j ∈ V

The induced function yields the label '1' for the edges e_2 ,e'_2 and e_{3m+1}**;** the label '0' for the edges e_1, e'_1 and e'_4**;** the label '2' for the edges e_3,e_5 and e'_3 **;**

for $0 \le i \le \lfloor (m-2)/3 \rfloor$, the edges e_{6+9i} receive label '0' ; the edges e_{4+9i} receive label '1' ; the edges e'_{5+9i} receive label '0' **;** the edges e'_{6+9i} receive label '2' **;**

for $0 \le i \le \lfloor (m-4)/3 \rfloor$, the edges e_{10+9i} receive label '0' ; the edges e'_{12+9i} receive label '1' ; the edges e'_{11+9i} receive label '2' **;** the edges e'_{10+9i} receive label '0' **;**

for $0 \le i \le \lfloor (m-4)/3 \rfloor$ and $0 \le j \le 1$, the edges $e_{11+9i+j}$ receive label '2' ;

for $0 \le i \le \lfloor (m-3)/3 \rfloor$ and $0 \le j \le 1$, the edges e_{8+9i+j} receive label '1' ;

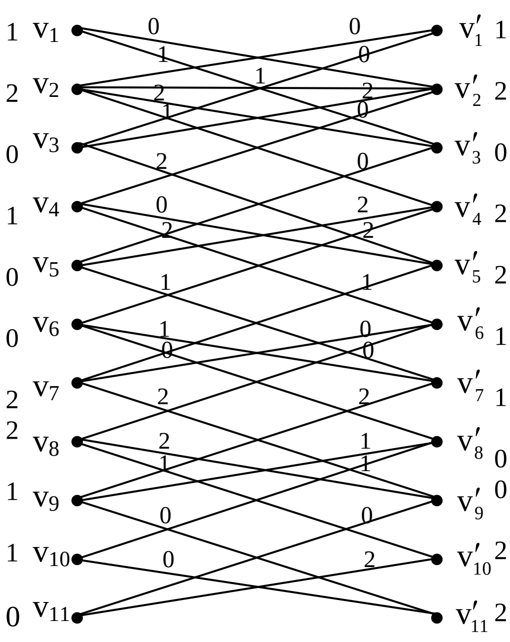
for $0 \le i \le \lfloor (m-5)/3 \rfloor$,the edges e_{14+9i} receive label '0' **;** the edges e'_{13+9i} receive label '1' **;**

for $0 \le i \le [(m-3)/3]$, the edges e_{7+9i} receive label '2' ; the edges e'_{8+9i} receive label '1' ; the edges e'_{9+9i} receive label '0' and the edges e'_{7+9i} receive label '2'.

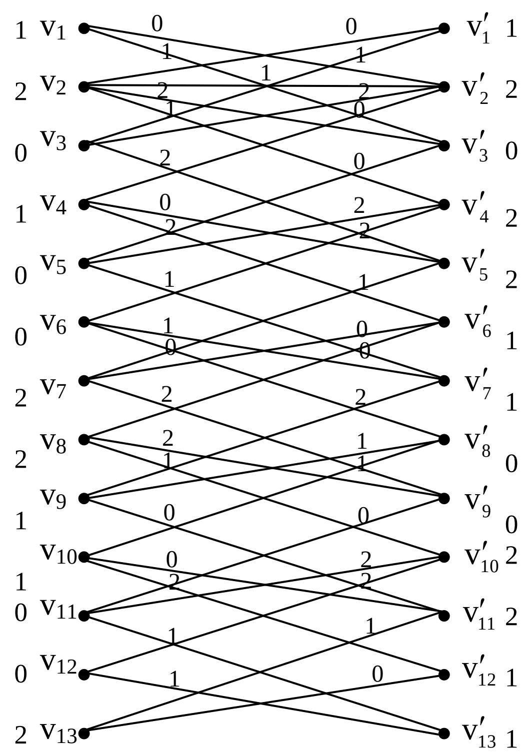
Thus all the 6m+1 edges are labeled.

Hence the extended duplicate graph of arrow graph A_m^2, $m \ge 2$ is total 3 sum cordial.

Illustration 3: Total 3 Sum Cordial labeling for the graphs **EDG (** A_5^2 **)** and **EDG (** A_6^2 **)**



EDG (A_5^2 **)**



EDG (A_6^2 **)**

3.2 E_3-CORDIAL LABELING

In this section, we present an algorithm and prove the existence of E_3 cordial labeling for the extended duplicate graph of arrow graph A^2_m , $m \geq 2$.

Algorithm: 3.2

Procedure [E_3 Cordial labeling for EDG (A^2_m), $m\geq 2$]

$V \leftarrow \{v_1, v_2, \dots, v_{2m}, v_{2m+1}, v'_1, v'_2, \dots, v'_{2m}, v'_{2m+1}\}$

$E \leftarrow \{e_1, e_2, \dots, e_{3m}, e_{3m+1}, e'_1, e'_2, \dots, e'_{3m}\}$

$e_2 \leftarrow 0$, $e'_1 \leftarrow 2, e'_2 \leftarrow 0, e'_3 \leftarrow 2$

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-2}{3} \right\rfloor$

$e_{5+9i} \leftarrow 2$

$e_{6+9i} \leftarrow 1$

$e_{1+9i} \leftarrow 1$

$e_{3+9i} \leftarrow 1$

$e_{4+9i} \leftarrow 0$

$end\ for$

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-3}{3} \right\rfloor$

$e_{9+9i} \leftarrow 2$

$e_{7+9i} \leftarrow 1$

$e_{8+9i} \leftarrow 0$

$end\ for$

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-4}{3} \right\rfloor$

$e_{11+9i} \leftarrow 0$

$end\ for$

$for\ \ i = 0\ \ to\ \left\lfloor \frac{m-2}{3} \right\rfloor$

$e'_{6+9i} \leftarrow 2$
end for
for $i = 0$ *to* $\left\lfloor \frac{m-2}{3} \right\rfloor$
for $j = 0$ *to* 1
$e'_{4+9i+j} \leftarrow 1$
end for
end for
for $i = 0$ *to* $\left\lfloor \frac{m-3}{3} \right\rfloor$
$e'_{9+9i} \leftarrow 2$
$e'_{7+9i} \leftarrow 2$
$e'_{8+9i} \leftarrow 0$
end for
for $i = 0$ *to* $\left\lfloor \frac{m-4}{3} \right\rfloor$
$e'_{12+9i} \leftarrow 2$
$e'_{11+9i} \leftarrow 0$
end for
for $i = 0$ *to* $\left\lfloor \frac{m-4}{3} \right\rfloor$
for $j = 0$ *to* 1
$e'_{10+9i+j} \leftarrow 1$
end for
end for
end procedure

Theorem 3.2: The extended duplicate graph of arrow graph A_m^2, $m \geq 2$ is E_3 cordial.

Proof: Let A_m^2, $m \geq 2$ be a arrow graph**.** In order to label the edges, define a function $f : E \rightarrow \{0,1,2\}$ as given in algorithm 3.2

The edges e_2,e'$_1$,e'$_2$ and e'$_3$ receive label '0' , '2' , '0' and '2' respectively **;**

for $0 \le i \le \lfloor (m-2)/3 \rfloor$, the edges e_{5+9i} receive label '2' ; the edges e_{6+9i} receive label '1'; the edges e_{1+9i} receive label '1'; the edges e_{3+9i} receive label '1'; the edges e_{4+9i} receive label '0'; the edges e'_{6+9i} receive label '2';

for $0 \le i \le \lfloor (m-3)/3 \rfloor$, the edges e_{9+9i} receive label '2' ; the edges e_{7+9i} receive label '1'; the edges e_{8+9i} receive label '0'; the edges e'_{7+9i} receive label '2' ; the edges e'_{8+9i} receive label '0' ; the edges e'_{9+9i} receive label '2';

for $0 \le i \le \lfloor (m-4)/3 \rfloor$, the edges e_{11+9i} receive label '0' ; the edges e'_{11+9i} receive label '0'; for $0 \le i \le \lfloor (m-2)/3 \rfloor$, the edges e'_{12+9i} receive label '2';

for $0 \le i \le \lfloor (m-2)/3 \rfloor$ and $0 \le j \le 1$,the edges e'_{4+9i+j} receive label '0' ;

for $0 \le i \le \lfloor (m-4)/3 \rfloor$ and $0 \le j \le 1$,the edges $e'_{10+9i+j}$ receive label '0'

Thus all the 6m+1 edges are labeled.

To obtain the labels for vertices, we define the induced function $f^* : V \rightarrow Z_3$ defined by $f^*(v_i) = \{\sum f(v_i, v_j) / v_i v_j \in E\} (\text{mod} 3)$

The vertices v_1, v_2, v_3, v'_1 , v'_2 and v'_3 receive label '1' , '0' , '1' , '2' , '1' and '2' respectively **;**

for $0 \le i \le \lfloor (m-2)/3 \rfloor$ and $0 \le j \le 1$, the vertices v_{4+6i+j} receive label '0' ;
for $0 \le i \le \lfloor (m-3)/3 \rfloor$ and $0 \le j \le 1$,the vertices v_{6+6i+j} receive label '2' ;
for $0 \le i \le \lfloor (m-4)/3 \rfloor$ and $0 \le j \le 1$,the vertices v_{8+6i+j} receive label '1' ;
for $0 \le i \le \lfloor (m-2)/3 \rfloor$,the vertices v'_{4+6i} receive label '1', the vertices v'_{5+6i} receive label '0' ;
for $0 \le i \le \lfloor (m-3)/3 \rfloor$, the vertices v'_{6+6i} receive label '0', the vertices v'_{7+6i} receive label '2';
for $0 \le i \le \lfloor (m-4)/3 \rfloor$,the vertices v'_{8+6i} receive label '1' ,the vertices v'_{9+6i} receive label '0'.Thus all the entire 4m+2 vertices are labeled.
Hence the extended duplicate graph of arrow graph A_m^2 , $m \ge 2$ is E_3 cordial.

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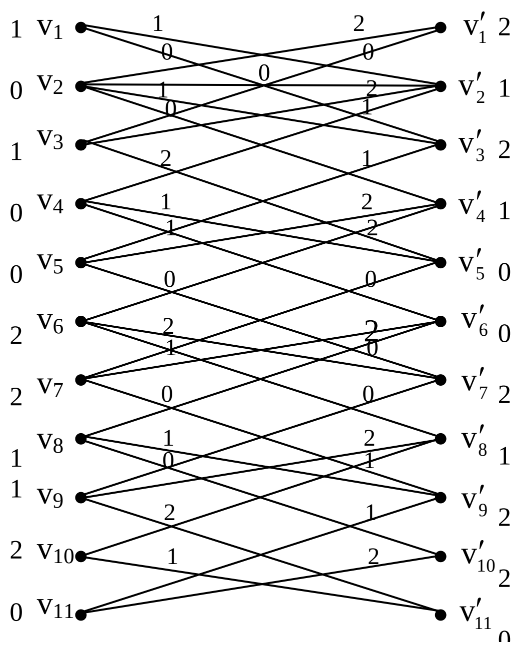
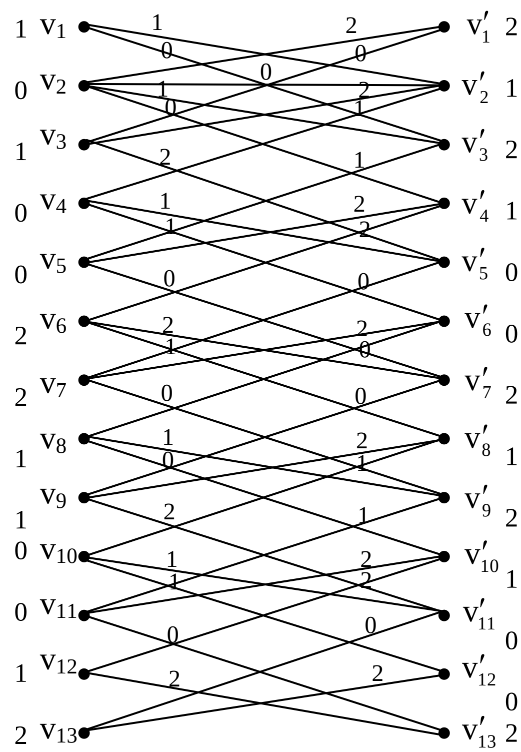
Illustration 4 : E_3 Cordial labeling for the graphs **EDG(**A_5^2**)** and **EDG(**A_6^2**)**



EDG (A_5^2**)**



EDG (A_6^2**)**

Theorem 3.3

The extended duplicate graph of arrow graph A_m^2 , $m \geq 2$ is total E_3 cordial.

Proof:

In theorem 3.2,

Case (i) for $m = 3n - 4, n \geq 2$

$m + \frac{m+1}{3}$ vertices were assigned the label "1", $m + \frac{m+1}{3}$ vertices were assigned the label "0" and $m + \frac{m+1}{3} + 1$ vertices were assigned the label "2". $2m + 1$ edges were assigned the label "1", $2m$ edges were assigned the label "0" and $2m$ edges were assigned the label "2".

Case (ii) for $m = 3n - 3, n \geq 2$

$m+\frac{m}{3}+1$ vertices were assigned the label "1", $m+\frac{m}{3}+1$ vertices were assigned the label "0" and $m+\frac{m}{3}$ vertices were assigned the label "2". $2m$ edges were assigned the label "1", $2m$ edges were assigned the label "0" and $2m+1$ edges were assigned the label "2".

Case (iii) for $m=3n-2, n\geq 2$

$m+\frac{m+2}{3}$ vertices were assigned the label "1", $m+\frac{m+2}{3}$ vertices were assigned the label "0" and $m+\frac{m+2}{3}$ vertices were assigned the label "2". $2m$ edges were assigned the label "1", $2m+1$ edges were assigned the label "0" and $2m$ edges were assigned the label "2".

In case (i), the number of vertices and edges labeled ''1'' is $m+\frac{m+1}{3}+2m+1$ $=\frac{10m+1}{3}+1$, the number of vertices and edges labeled ''0''is $m+\frac{m+1}{3}+2m$ $=\frac{10m+1}{3}$ and the number of vertices and edges labeled ''1'' is $m+\frac{m+1}{3}+1+2m=\frac{10m+1}{3}+1$, which differ by at most one.

In case (ii), the number of vertices and edges labeled ''1'' is $m+\frac{m}{3}+1+2m=\frac{10m}{3}+1$, the number of vertices and edges labeled ''0'' is $m+\frac{m}{3}+1+$ $2m$ $=\frac{10m}{3}+1$and the number of vertices and edges labeled ''1'' is $m+\frac{m}{3}+2m+1$ $=\frac{10m}{3}+1$, which differ by at most one.

In case (iii), the number of vertices and edges labeled ''1''is $m+\frac{m+2}{3}+2m$ $=\frac{10m+2}{3}$, the number of vertices and edges labeled ''0'' is $m+\frac{m+2}{3}+2m+1$ $=\frac{10m+2}{3}+1$and the number of vertices and edges labeled ''1'' is $m+\frac{m+2}{3}+2m$

$= \frac{10m+2}{3}$, which differ by at most one and satisfies the required condition. Hence the extended duplicate graph of arrow graph A_m^2, $m \geq 2$ is total E_3 cordial labeling.

4. Conclusion

In this paper, we presented algorithms and proved that the extended duplicate graph of arrow graph A_m^2, $m \geq 2$ admits total 3-sum cordial , E_3- cordial and total E_3 - cordial labeling .

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